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Straight and Crooked Uses of Electrical Resistances

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Abstract: Elementary combinations of electrical resistances in varieties of form are taken into consideration and the corresponding equivalent resistances with the components at rest and also in motion along with their effects on the electrical flow in normal electrical circuits are ventured. This work is partly an undergraduate
physics project.

Key Words: Resistances, Circuits, Equivalent Resistance, Electric Current

I. INTRODUCTION

The fact and phenomena of obstruction to electric current is broadly called electrical resistances. Its elementary form comes from Ohm's law which is valid only for conductors. For conductors then resistance is a parameter equals to the ratio of emf or potential difference and electric current;

$R = \frac{V}{I}$

From free electron theory of metals for ordinary physical situation it can be shown
that $R = \rho\{L/A\}$ where $\rho = \{(me^2)/(n\tau)\}$ where R , V , I, ρ , L A , m , e , n and τ are respectively the resistance, potential difference, electric current intensity ,resistivity, length ,cross-sectional area, electronic mass, electronic charge , number-density of free electrons and mean-free-flight time of free electrons.

Various facts and phenomena are there within conducting materials comprising the internal statics and dynamics of electrical resistances in general which might be dealt with in another similar article as this one in future.In this article various elementary and some complex combinations of electrical resistances are considered and the consequences of secular and transient contacts with them are explored.

Study: Case(i): When the series-equivalent is n-times the parallel-equivalent for two different
Resistances R_1 and R_2 .

$R_1 + R_2 = \frac{n(R_1R_2)}{R_1+R_2}$; or,

$R_1 = \frac{(n-2)R_2 \pm \sqrt{\{(2-n)^2R_2^2 - 4R_2^2\}}}{2}$; For R_1 and R_2 being positive real

numbers $n \geqq 4$ has to be satisfied. Therefore n is always greater than or equal to '4'.

Case(ii)

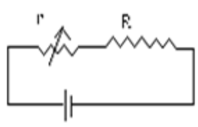

Fig.1

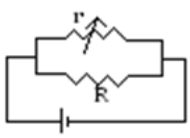

Fig.2

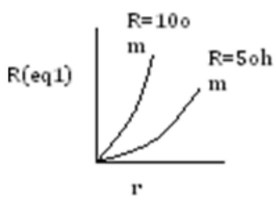

Fig.3

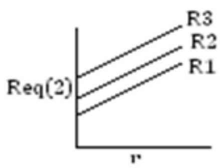

Fig4

When one of the component-resistances is variable the variation of equivalent resistance in case of parallel combination is nonlinear one while in case of series combination it is linear.
When both of the two component-resistances increase the equivalent resistance increases In both case similarly.

Case(iii) When one of the pair of resistances increases and the other decreases the equivalent resistance is found to vary in the following way;

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$R^{p}_{eq.} = \frac{R_1 R_2}{R_1+R_2} \approx \frac{R_{10}R_{20}\{1+(\alpha_1-\alpha_2)z\}}{(R_{10}+R_{20})+(R_{10}\alpha_1-R_{20}\alpha_2)z}$ where z is a physical factor such as temperature etc. and α's are that factor- coefficient of variation of resistance and are usually taken to be very small such that

$\alpha_i\alpha_j \approx 0.$

$$R^{s}_{eq.} = R_1 + R_2 = (R_{10}+R_{20}) + (R_{10}\alpha_1 - R_{20}\alpha_2)z$$

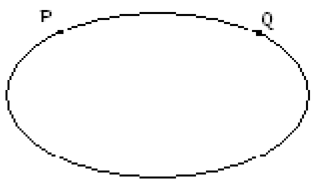
$$\text{If , } R_{10} = R_{20} = R_{00} \qquad R^{p}_{eq.} = \frac{R_{00}\{1+(\alpha_1-\alpha_2)z\}}{2+(\alpha_1-\alpha_2)z} \quad \Rightarrow \quad R^{p}_{eq.} < R_{00}$$

$$R^{s}_{eq.} = R_{00}\{2+(\alpha_1-\alpha_2)z\} \quad \Rightarrow \quad R^{s}_{eq.} > R_{00}$$

$$\text{for } \alpha_1 > \alpha_2 > 0 \quad \forall \quad \alpha_1 < \alpha_2 \;\wedge\; (\alpha_2-\alpha_1)z < 2$$

z' is either time or any other independent variable which the magnitude of component resistances $R_1 and\ R_2$ does depend on. We see here that $R^{p}_{eq.}$ decreases always for one resistance decreases and the other increases. But $R^{s}_{eq.}$ may increase or decrease for the same condition depending on the relative values of $\alpha_1 and\ \alpha_2$.

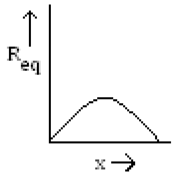
Case(iv) Let us consider a closed loop of wire with a pair of contact (P & Q)on it. Suppose the



P total length of the closed loop is 'L' and 'x' be its part between two points. Then the equivalent resistance between P and Q will be given by

$$R_{eq} = \frac{\sigma x\,(L-x)}{L} \quad \text{where}$$

σ = Resistance per unit length = $(\frac{R}{L})$. There is a maximum value of R_{eq} at $x = (\frac{L}{2})$ and is '$(\frac{R}{4})$'

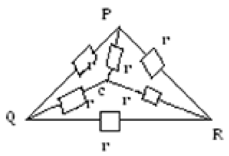


From the above equation it may be written for a constant total current I in the loop as

$({}^{V}/_{I}) = \sigma\left\{{}^{x(L-x)}/_{L}\right\}$ and consequently $[{}^{d^2V}/_{dx^2}] = -\frac{2\sigma I}{L}$

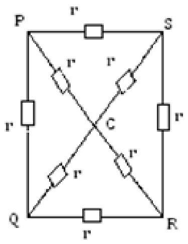
Case(v) Two dimensional circuitry with resistances must fall in any one of the following categories of combination of resistances ;namely (i)Series , (ii) Parallel , (iii) Series-parallel combined or mixed ,(iv)Complex combination. The simplest version of complex combination is what is known as Wheatstone-bridge. And in all of these cases equivalent resistances can be found out by proper application of either T-π network conversion or of Kirchoff's laws and by careful calculations thereby. Some examples are given below;(These circuit-elements may be called 'web-network' of resistances).

A)



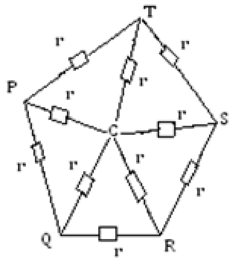
R^{CP}_{eq} = 0.5r , R^{QP}_{eq} = 0.5r

B)



R^{CP}_{eq} = (7/15)r = 0.467 , R^{QP}_{eq} = (8/15)r = 0.533

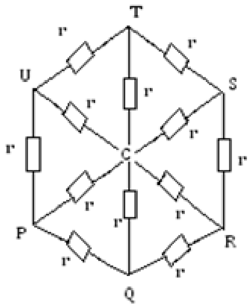
C)



$R_{eq}^{CP} = (5/11)r$ =0.45455 r , $R_{eq}^{QP} = (6/11)r$ = 0.54545 r

$R_{eq}^{PR} = (8/11)r$ = 0.72727

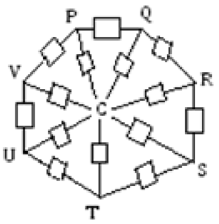
D)



$R_{eq}^{CP} = (9/20)r$ =0.45 r , $R_{eq}^{QP} = (11/20)r$ = 0.55 r

$R_{eq}^{PR} = (3/4)r$ =(15/20)r =0.75 r , $R_{eq}^{PS} = (4/5)r$ =(16/20)r =0.8

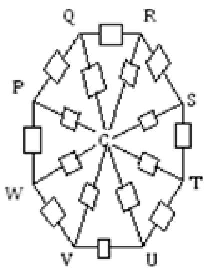
E)



$R_{eq}^{CP} = (13/29)r$ =0.4483 r , $R_{eq}^{QP} = (16/29)r$ =0.5517 r ,

$R_{eq}^{PR} = (22/29)r$ =0.7586 r , $R_{eq}^{PS} = (24/29)r$ =0.8276r

F)



$R_{eq}^{CP} = (47/105)r$ =0.4476 r , $R_{eq}^{QP} = (58/105)r$ =0.5524 r

$R_{eq}^{PR} = (80/105)r$ =0.7619 r , R_{eq}^{PS} =(88/105) r =0.8381r

$R_{eq}^{PR} = (90/105)r$ =0.8571 r

Table-I

(Number of Arms versus Equivalent Resistances)

N= / R_{eq}	3	4	5	6	7	8
R_{eq}^{CP}	0.5000 r	0.4670 r	0.45455 r	0.4500 r	0.4483 r	0.4476 r
R_{eq}^{QP}	0.5000 r	0.5330 r	0.54545 r	0.5500 r	0.5517 r	0.5524 r
R_{eq}^{Total}	1.0000 r	1.0000 r	1.00000 r	1.0000 r	1.0000 r	1.0000 r

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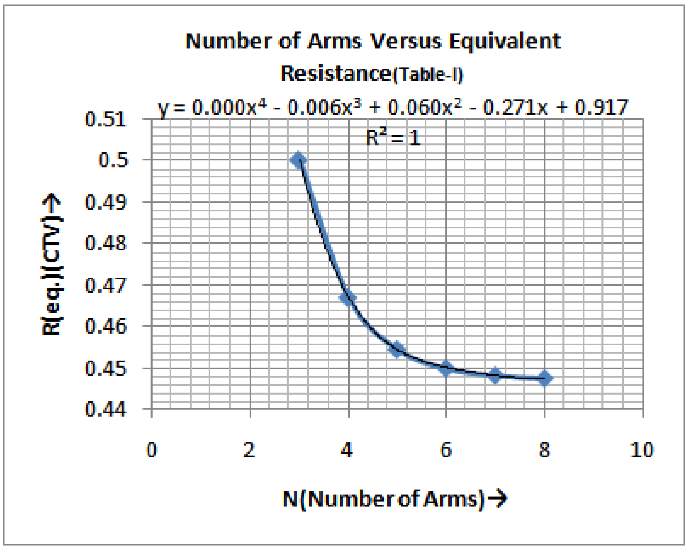
Table-II

(Arm-span versus Equivalent Resistance with total number of arms of Polygon as parameter)

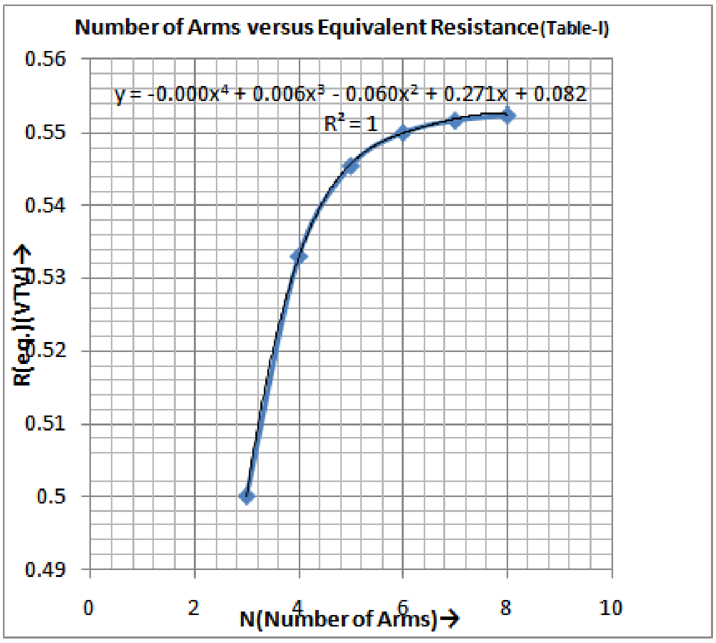
N= / R_{eq}	3	4	5	6	7	8
R_{eq}^{CP}	0.5000 r	0.4670 r	0.45455 r	0.4500 r	0.4483 r	0.4476 r
R_{eq}^{PQ}	0.5000 r	0.5330 r	0.54545 r	0.5500 r	0.5517 r	0.5524 r
R_{eq}^{PR}	0.5000 r	0.6667 r	0.7273 r	0.7500 r	0.7586 r	0.7619 r
R_{eq}^{PS}	X	0.5330 r	0.7273 r	0.8000 r	0.8276 r	0.8381 r
R_{eq}^{PT}	X	X	0.54545 r	0.7500 r	0.8276 r	0.8571 r



R(eq.)(CTV)⇒$R_{eq.}^{CP}$

CTV⇒Center to Vertex

Fig.1



R(eq.)(VTV)⇒$R_{eq.}^{QP}$

VTV⇒Vertex to Vertex

Fig.2

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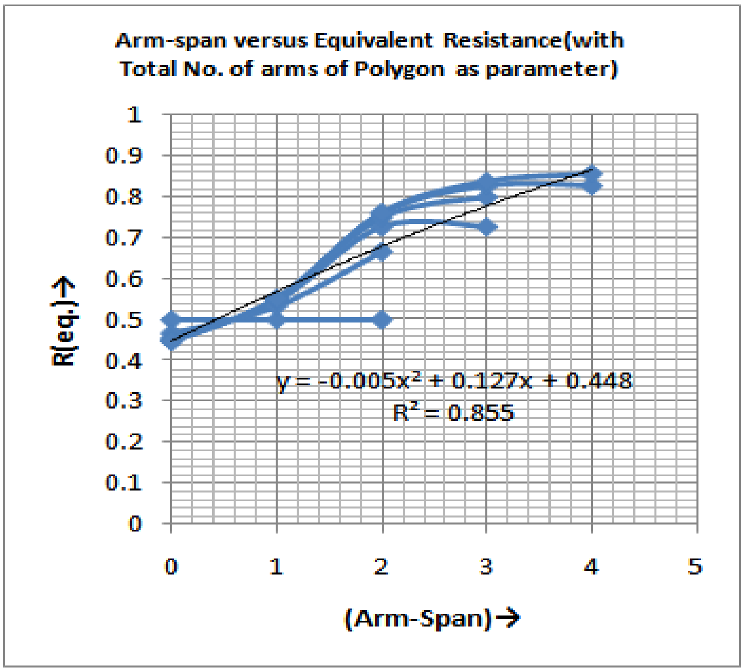
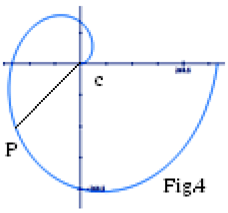


Fig.3

From the Table-I given above one can get a theorem which states that ***"For a Closed web-Network of resistance $R^{CV}_{eq.}$ + $R^{VV}_{eq.}$ = 1 when $R^{CV}_{eq.}$ refers to equivalent resistance between center-to-vertex and $R^{VV}_{eq.}$ refers to equivalent resistance between vertex-to-next vertex."***
One most significant application of this theorem is in case of all equal component resistances is the uniqueness of definition of ***'absolute unit of resistance'.*** This theorem may also perhaps be generalized for any polygonal network of resistances.

Case(VI): For a ***spiral Network of continuous medium resistance*** the equivalent resistance is found to vary with only a single variable and that is the angular coordinate of the terminal-point.The equivalent resistance between the central initial point 'C' and the final terminal point 'P' is to be found out.Here the whole spiral is a continuous resistor and starting from a central point C the resistor rolls out and one is to calculate the equivalent resistance between C and any arbitrary point 'P'on the stretch of the spiral arm of the resistor.The simple spiral has a standard equation $r = a\varphi$ where $'a'$ is a positive constant and 'φ'' is the angular coordinate of the point 'P' along with 'r' being the radial distance from 'C', φ being expressed in radian. The equivalent resistance 'R^{CP}_{eq}'' = σ r = σ $a\varphi$ where 'σ' is resistance per unit length of



the spiral resistor. But measuring the equivalent resistance between the points 'C' and 'P' needs a contact with another conductor between 'C' and 'P' which has also certain amount of resistance. That means its resistance being considered the system becomes a parallel combination of resistances and the ultimate equivalent resistance becomes

$$R^{CP}_{eq\,(F)} = \{\frac{R^{CP}_A\, R^{CP}_L}{R^{CP}_A + R^{CP}_L}\} = \frac{\sigma a\,\theta^2}{(2+\theta)}\,.$$

Here in this figure(Fig.4(a)) y' represents $'R^{CP}_{eq\,(F)}\,'$ and x' represents 'θ' of the above equation. Obviously the whole negative part of the graph on the left of origin is extraneous while the positive part only on the right side of the origin gives one the real values showing also the minimum value equals to zero at 'θ = 0' and obtain a real plausible appreciable value much greater than unity

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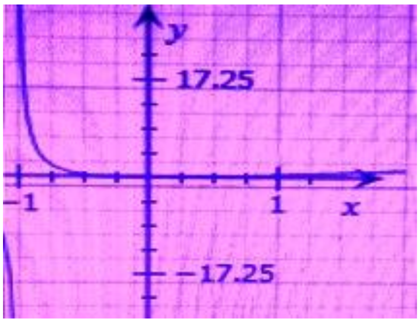

Fig.4(a) even for 'θ' becomes infinity though continuously increasing function i,e. an asymptotic approach.

Case(vii): Three-dimensional resistance-circuit;

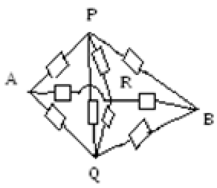

Fig.5

The network shown aside may be called a real three dimensional network because it is impossible to restructure this into a planner network just by means of geometrical transformation. But with the application of π-T conversion-factor one can do that as follows;

Then one can find three T-networks in this configuration which is then again to be transformed into corresponding π-network and they are respectively (CD-AD-BD), (AF-BF-CF), (AE-CE-DE).

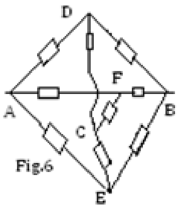

Fig.6

All A-B-equivalent from T to π will be given by

$$R_{AB} = \frac{r.r}{(\frac{r}{3})} + r + r = 5\,r$$

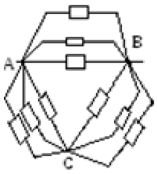

Fig.7

All A-C-equivalent from T to π will be given by $R_{AC} = \frac{r.(\frac{r}{3})}{(\frac{r}{3})} + r + \frac{r}{3}$

$= \frac{5r}{3}$ and similarly $R_{BC} = \frac{5r}{3}$

Then the total equivalent resistances across AC, BC, & AB will be given by

$R_{ABT} = \frac{5r}{3}$, $R_{ACT} = \frac{5r}{9}$, $R_{BCT} = \frac{5r}{9}$.

Then the net effective equivalent resistance across AB is given by

$$R_{eq} = \{ R_{ABT}(R_{ACT} + R_{BCT}) / (R_{ABT} + R_{ACT} + R_{BCT}) \} = \frac{\frac{5r}{3} X \frac{10r}{9}}{\frac{25r}{9}} = \frac{2r}{3}.$$

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Another example of a three-dimensional resistance-network may be considered as the Following one;

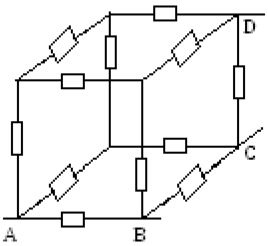


Fig.9

Equivalent resistance between (i) any two adjacent points(for example A,B) (ii) two planner diagonal points(for example A,C), (iii) two non-coplanar diagonal points (for example A,D) may be found by T-π and also π-T transformation at least for 10 to 12 steps in each case(Ref.). The values of equivalent resistances for the three cases as mentioned here will thus be respectively ,

$$R_{AB} = \frac{7r}{12}$$
$$R_{AC} = \frac{3r}{4} = \frac{9r}{12}$$
$$R_{AD} = \frac{5r}{6} = \frac{10r}{12}$$

Whichever be the number of arms of a poly-gonal circuit with resistance the ultimate equivalent resistance between any two vertices however long the circuit may be is always less than a single resistance. If we assign a number '0' to the vertex 'A' and 'n' to another arbitrary vertex in the same network 'I' (say) then $R_{0n} < r$ for an n-gon closed network-circuit with all equal resistance-arms each of resistance 'r' and then $R_{0\infty} \approx r$.

<u>**The Dynamical Resistance-combination:**</u> ***Case(viii) Resistance Comb:*** A solid conductor with a large number ,say N ,of resistance-arms in the form of spikes of same common height are firmly attached and positioned on the conductor at a regular interval 'a' (say) constituting a parallel array of resistances is called a Resistance-Comb. This resistance-comb along with a moving connector(C in Fig.10) moving with a uniform velocity 'v' (say) yields results of some interest. Let us for this case in particular consider all the resistances in the parallel array in the comb be of same value 'r' while the fixed primary resistance in the circuit be 'R_0' . 'E' is the constant emf in the circuit ,G the galvanometer to measure the current ,K the key. The connector is presumed to run very smoothly over the arms' edge . At any time 't' if the connector connect 'n' number of arms then for the instantaneous current in the circuit one can write the following;

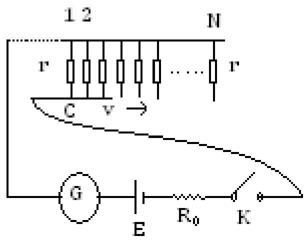


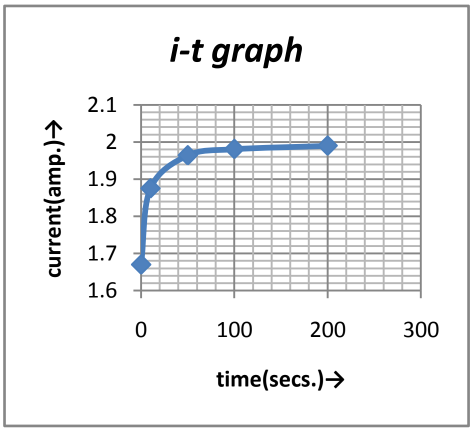
Fig.10

$$i = \frac{E}{(R_0 + \frac{r}{n})} = \frac{E}{R_{eq}}$$, R_{eq} being a variable equivalent resistance due the moving connector .

$$\frac{dR_{eq}}{dt} = -\frac{1}{n^2}\left(\frac{dn}{dt}\right) \quad \text{where} \quad \frac{dn}{dt} = \frac{v}{a}$$. Then $n = n_0 + (\frac{v}{a})t$. For this case let $n_0 = 1$.

Then one can write $n = 1 + (\frac{v}{a})t$ and consequently

$$i = \frac{E}{(R_0 + \frac{r}{1 + (\frac{v}{a})t})}$$ giving $i = i_{min} = \frac{E}{R_0 + r}$ at $t = 0$ and $i = i_{max} = \frac{E}{R_0 + \frac{r}{N}}$ at $t = \tau$(say).



The nature of variation of current with time is demonstrated through graph(Fig.11) where for an example the following set of values of the parameters have been taken: E =10 v , R_0= 5Ω , r = 1Ω , $\frac{v}{a}$ = 0.2

i_{max} = 2 amps. at t =∞ . But a less maximum value for $n = N$ can be found out for known value of N. For different set of values of all the parameters concerned chosen appropriately different graphical points may be

Fig.11

(11)

obtained keeping the over all nature of the graph exactly same as given here. As can be observed here that the rate of current is extremely slow and this slowness can be controlled controlling the value of 'v' and changing the set of values of the

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parameters too. Extremely slow change of small current may be required in some type of experiments both in microscopic and macroscopic level for various scientific investigation where quasi-equilibrium states are involved.

Now if instead of slow unidirectional motion of the connector it starts executing SHO given by $x = A\ \sin \omega t$ then the changes are as follows;

$$\frac{dn}{dt} = \frac{\omega A \cos \omega t}{a} \quad \text{where } 'n' \text{ is given by } n = 1 + \left(\frac{A}{a}\right) \sin \omega t \ ;$$

Then i as a function of time may be expressed as follows:

$$i = \frac{E}{R_0 + \frac{r}{1+\left(\frac{A}{a}\right)\sin\ \omega t}}$$

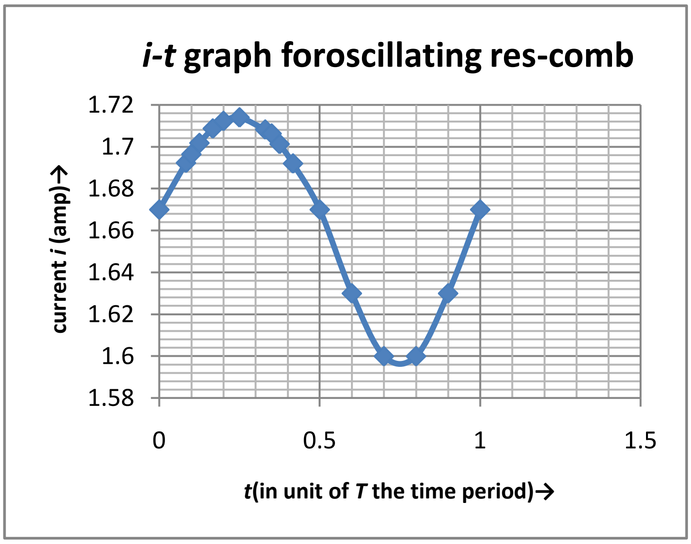

Fig.12

It is here interesting to note that the current derived from a steady voltage supply source becomes oscillatory just by means of an oscillating resistance-comb. Therefore this device may be used as a converter which converts dc to oscillatory dc as the oscillatory current though positive which may be made an ac just by biasing this with a suitable negative steady voltage. This is probably a very useful application of the oscillating resistance-comb circuit. The point to be mentioned here that without the help of any external magnetic field and use of rotor the ac is being produced.

Case (ix):The circular resistance comb with a rotor:

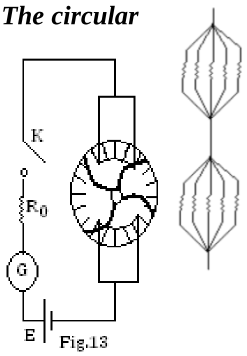

Fig.13

The model circuit is shown in Fi.13 and aside of it is being shown the equivalent resistance of the circuit. From this it easy to guess how the resistance of the rotating circular resistance comb works for an instant and then the effect of rotation of this will have to be considered. If all the value of resistances in all the arms are the same then there will be nothing interesting in it except some intermittent gap in contact. But if the value of the resistances in the consecutive arms are of some increasing order like following AP or GP then the case will on one hand be interesting while on the other hand be a little bit cumbersome as regards to the calculations are concerned. Here an orderly array of mutually identical resistance arms uniformly firmly fixed on the inner-side of a circular ring that can be rotated externally while the inner top head of a certain number of arms on both side of the center of the ring are connected through a wire-mesh to terminals of an electrical circuit comprises what is called a Rotor-Resistance-Comb. External rotor will make resistance-wheel-comb rotate with a definite angular speed of rotation 'ω' (say). Here the net equivalent resistance in the circuit is given by,

$$R_{eq} = R_0 + (R_1 + R_2) \quad \text{while} \quad R_1 = \left[\sum_{i=l}^{m} \frac{1}{R_i}\right]^{-1} , \quad R_2 = \left[\sum_{j=\frac{N}{2}+l}^{\frac{N}{2}+m} \frac{1}{R_j}\right]^{-1} \quad \text{where 'N' is}$$

the total even number resistance-arms in the wheel comb. The 't_0' which is common period of

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time for each step-change of R_{eq} is given by

$t_0 = \frac{1}{N\nu}$ where 'ν' is the frequency of rotation of the wheel-comb. The time rate of change of net equivalent resistance of the circuit is given by

$$\frac{dR_{eq}}{dt} = \frac{d(R_1+R_2)}{dt} = \frac{\Delta(R_1+R_2)}{t_0} = N\nu\, \Delta(R_1 + R_2).$$

For an example of how does the output from a simple circuit with Rotor-Resistance-wheel-Comb transforms(Fig.13) the following set of values of the different physical quantities are considered; N= 100 , ν = 50 Hz , $R_i = i\ \Omega$ where 'i' is simple integral natural number starting from i=1 to i= 100 with $m - i = 3$ i.e. for 3-arms' connection at a time on one side and $R_0 = 10\Omega$. The result obtained by putting this value in appropriate equation given above and numerical calculations thereafter is representationally depicted in the graphs (Fig.14 & Fig.15). As can be observed from the graphs that the output from a steady dc supply in a simple circuit becomes a periodic step-function-like entity that may be regarded as a transformed saw-tooth wave. Varying the values of the parameters concerned different other type of variations may be obtained as output result and input steady current may undergo varieties of transformation which may be required to yield different response pulses expected to serve the purpose in different field of technological applications.

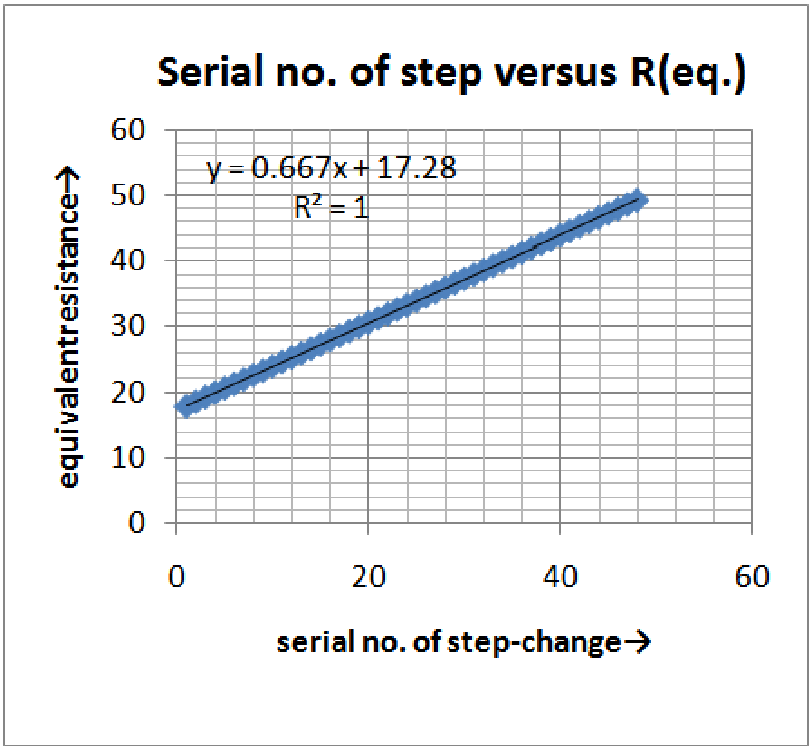

Fig.15

II. CONCLUSION:

Finding out equivalent resistance is a simple exercise in undergraduate course.

When the resistance-network under consideration is a complex one Kirchoff's laws are required to find out the component resistances or the current in any branch of a mesh. But when the complex network of resistances gets more and more complicated star-delta transformationknown also by an alternative name "T-π " transformation helps too much to find equivalent resistance and thereby any consequential values such as as that of branch-current, potentialdrop across varieties of complex combinations are considered here and equivalent resistances in each such case are found out revealing the way one can do that for any such case with anyother values of resistances. There is a difficulty in resolving a problem with moving resistance comb and that is the mathematical expressions considered here are continuous functions of time while the realphysical variation of the output current should be somewhat a step-like function of time corresponding to the common periodic gap 'a' between each pair of consecutive arms.

Complexity arises in generating such a function which can give the real physical variation with exactitude. Perhaps Kr$ö$enecker-delta may help in this regard. More complications arise if the values of the resistances in the arms are sequentially increasing by a fixed small margin.The required mathematical function will be much more complex yet the result perhaps will be more interesting and perhaps useful to Biomedical and Biotechnological research including some other areas of small-scale variations. However the models as demonstrated and explained here are overall useful and will be working for various purpose as average variation over the total period of motion will not differ very much after all. Moreover this is completely an analogue system of operation which even to some extent may be manually controlled. And lastly the model of rotating resistance-wheel with a finite number of arms on the wheel-brim at inner side ,each arm with resistance in an increasing order following a sequence produces a periodic step-function like variation from a steady input to the circuit. Mathematical complexities are involved in this case too which can perhaps be overcome with the help of superfast computer used today. Moreover more study is required for the model even by varying the angular speed in different order for the model to be useful in various technological fields of applications.

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